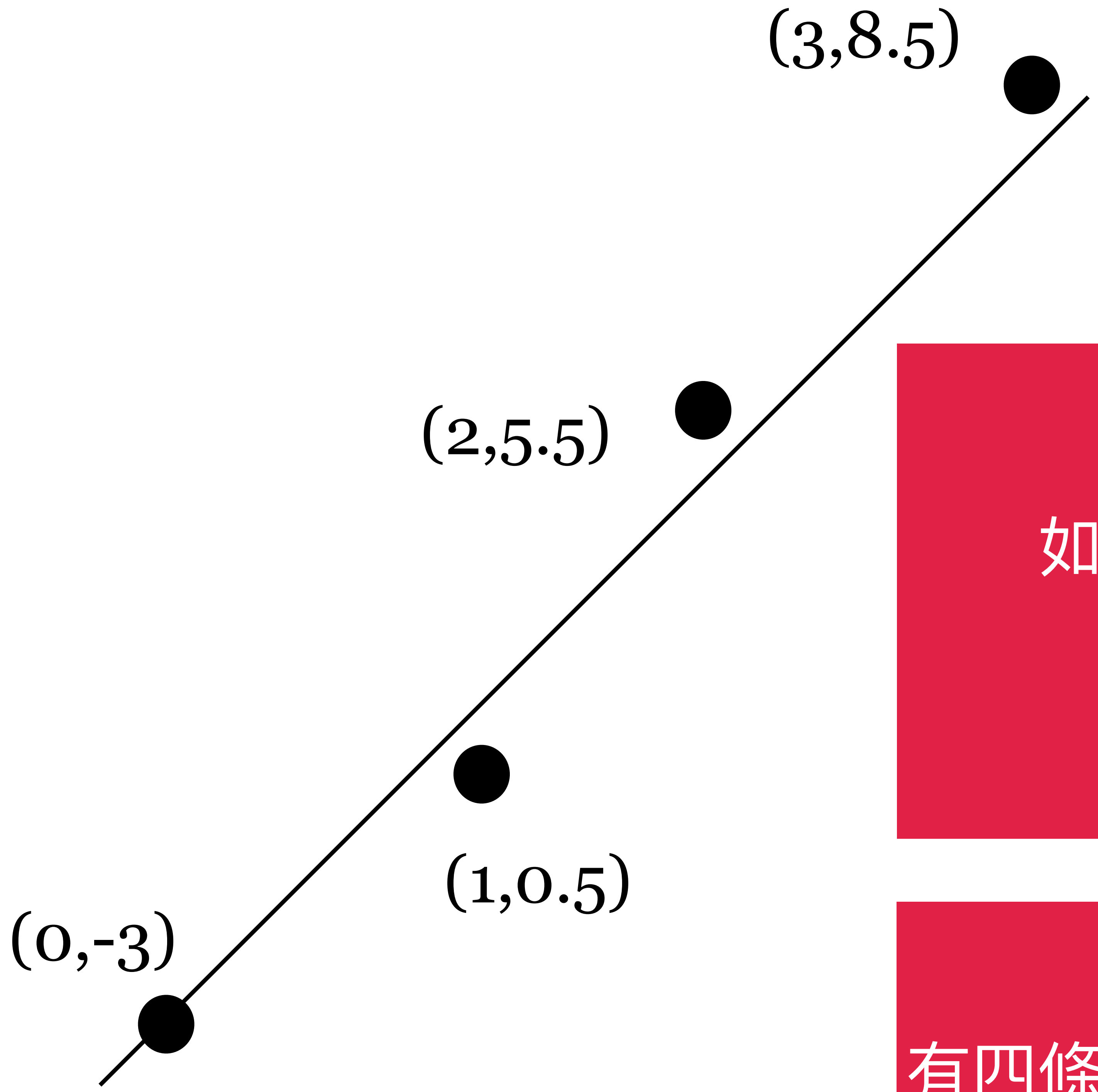
反矩陣的資料分析應用

給不共線的多個點求一直線



四點不共線
如何求出最好的一條線，
描述這四點呢？

線性系統是長方形的
有四條方程式，但是只有兩個未知數

步驟一：匯入套裝

```
import numpy as np
from numpy.linalg import inv
```

步驟二：將資料以矩陣A向量b表示

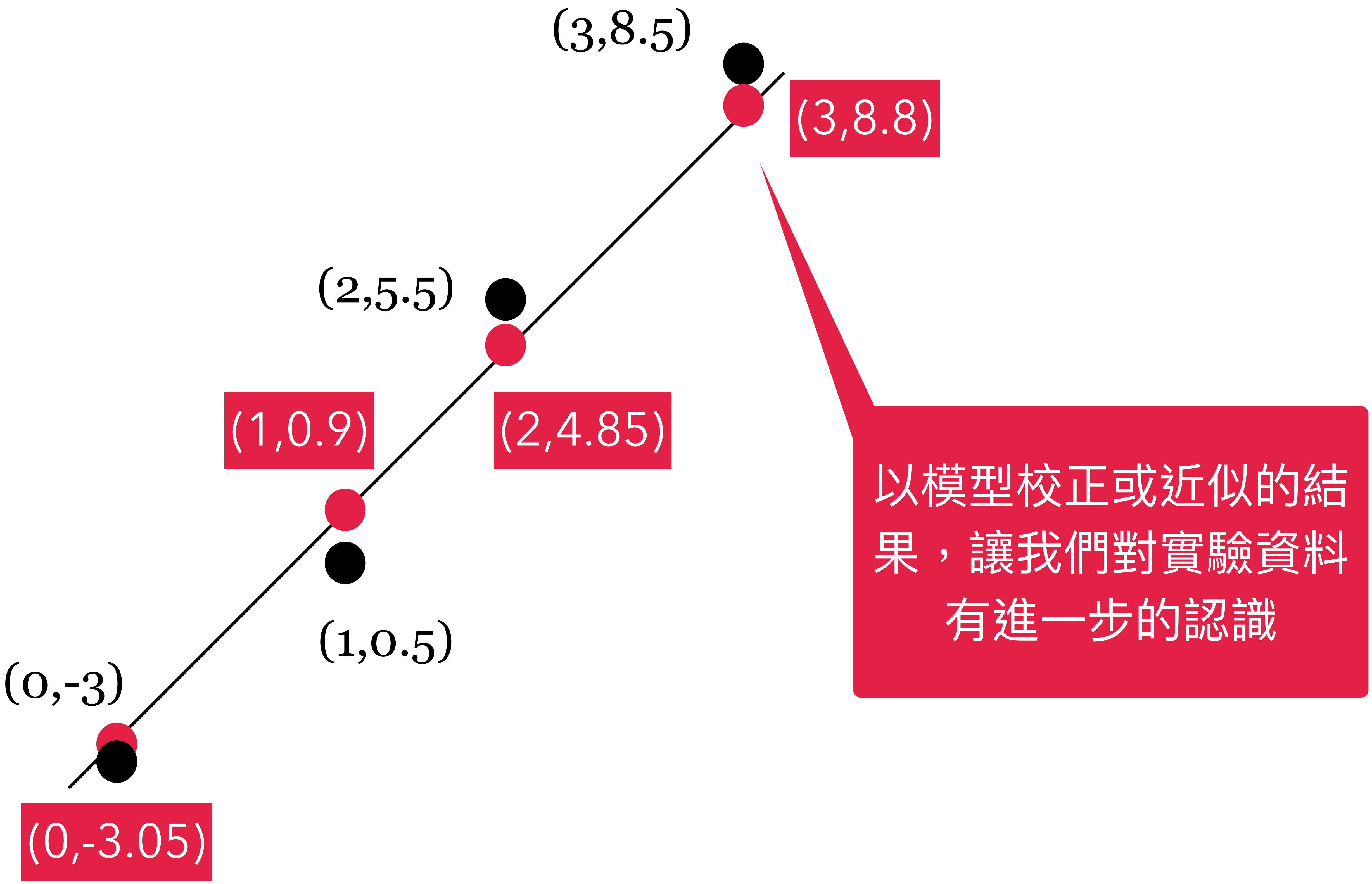
```
A = np.matrix([[2,1],[1,1],[3,1],[0,1]])
b = np.matrix([[5.5],[0.5],[8.5],[-3]])
```

步驟三：求轉置矩陣

```
AT = np.matrix.________(A)
```

步驟四：求最佳參數

```
B = ______ A
invB = ___(B)
ans = ______________
```



(3,8.5)
(3,8.8)
(2,5.5)
(1,0.9)
(2,4.85)
(1,0.5)
(0,-3)
(0,-3.05)
以模型校正或近似的結果，讓我們對實驗資料有進一步的認識

```python
import numpy as np
from numpy.linalg import inv
```

Matrix

```
A = np.matrix([range(1,5)])
```

```
matrix([[1, 2, 3, 4]])
```

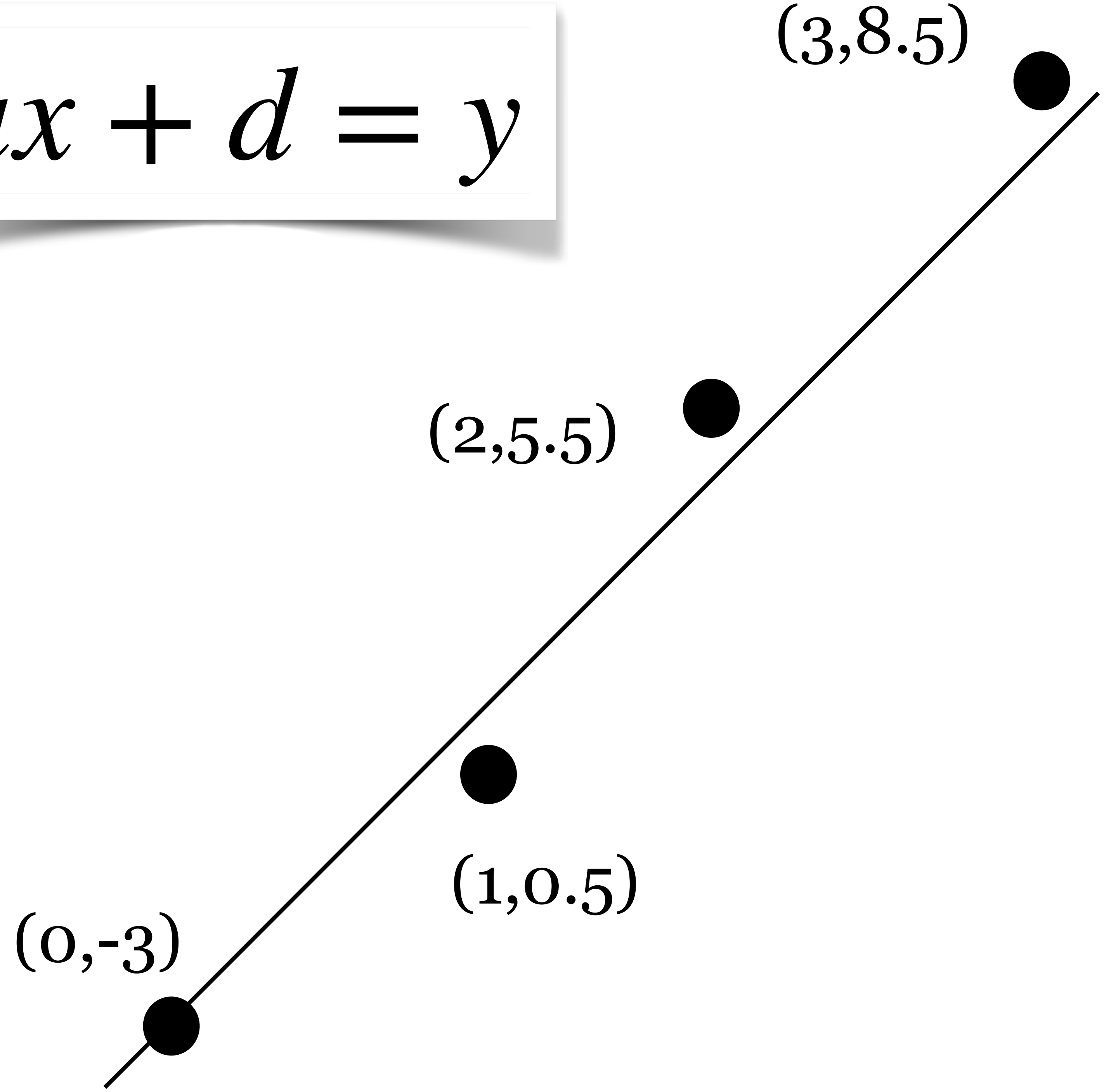
轉置矩陣

Matrix 轉置

```
A = np.matrix([range(1,5)])
np.matrix.transpose(A)
```

```
>>> np.matrix.transpose(A)
matrix([[1],
        [2],
        [3],
        [4]])
```

$$ax + d = y$$



$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = y$$

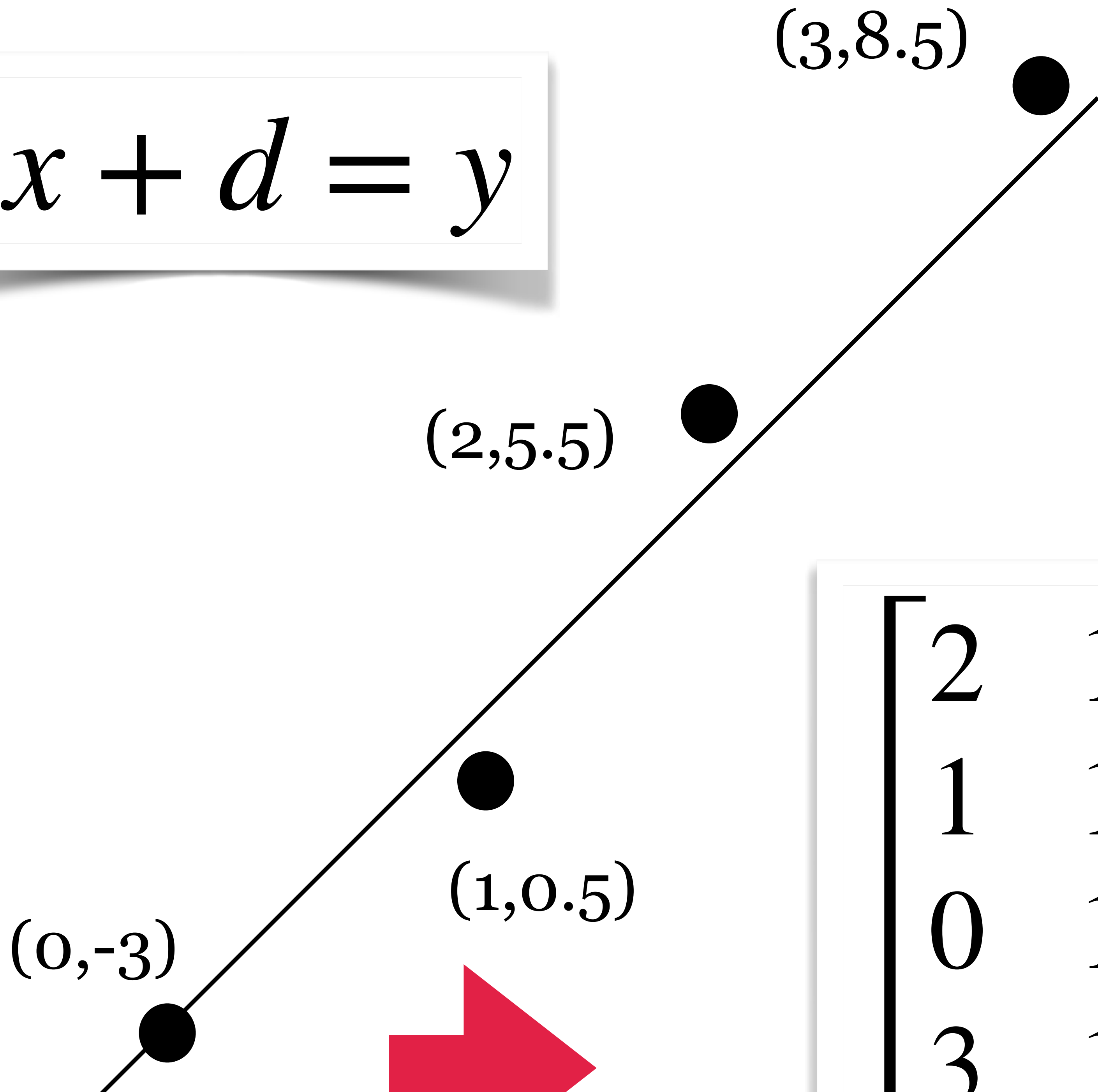
$$\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = 5.5$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = 0.5$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = -3$$

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = 8.5$$

$$ax + d = y$$



$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

長方形的系統

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

矩陣A

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

向量b

```
A = np.matrix([[2,1],[1,1],[3,1],[0,1]])
b = np.matrix([[5.5],[0.5],[8.5],[-3]])
```

```
>>> A
matrix([[2, 1],
        [1, 1],
        [3, 1],
        [0, 1]])
```

```
>>> b
matrix([[ 5.5],
        [ 0.5],
        [ 8.5],
        [-3. ]])
```

```
A = np.matrix([[2,1],[1,1],[3,1],[0,1]])
b = np.matrix([[5.5],[0.5],[8.5],[-3]])
```

可以求A的反矩陣嗎？

反矩陣一定是正方形的矩陣才有反矩陣

使用轉置矩陣
讓等號左邊變成
正方形矩陣

A^t A A^t b

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

```
A = np.matrix([[2,1],[1,1],[3,1],[0,1]])
b = np.matrix([[5.5],[0.5],[8.5],[-3]])
AT = np.matrix.transpose(A)
AT @ A
```

```
>>> print(AT@A)
[[14  6]
 [ 6  4]]
```

```
>>> AT @ b
matrix([[37. ],
        [11.5]])
```

$$\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

```
>>> print(AT@A)
[[14  6]
 [ 6  4]]
```

```
>>> AT @ b
matrix([[37. ],
        [11.5]])
```

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

可以使用反矩陣嗎？

等號左邊已經是正方形矩陣
那就可能有反矩陣

觀察：向量元素不同

討論：d不是只代表截距

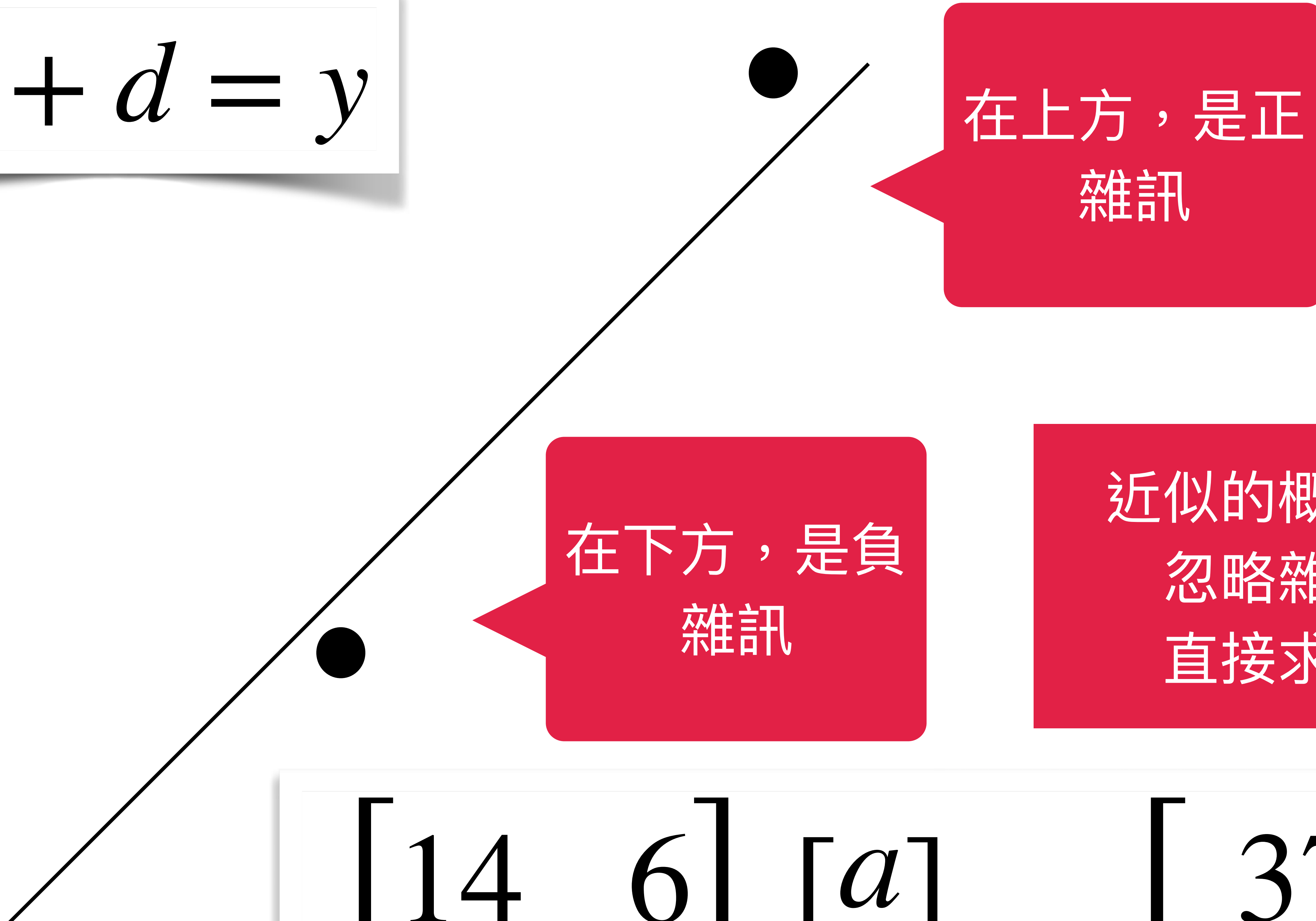
那代表時麼呢？

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

*d*代表截距+雜訊
因為點不在直線上

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

$$ax + d = y$$



近似的概念：
忽略雜訊
直接求解

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

最小（平方）近似誤差
提供近似概念

可以求最小（平方）近似誤差的直線

近似的概念：
忽略雜訊，直接求解

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ d \end{bmatrix} = \boxed{inv\left(\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix}\right)\begin{bmatrix} 37 \\ 11.5 \end{bmatrix}}$$

```
>>> A
matrix([[14,  6],
        [ 6,  4]])
```

```
B = np.matrix([[14,6],[6,4]])
invB = inv(B)
ans = invB @ np.matrix([[37],[11.5]])
```

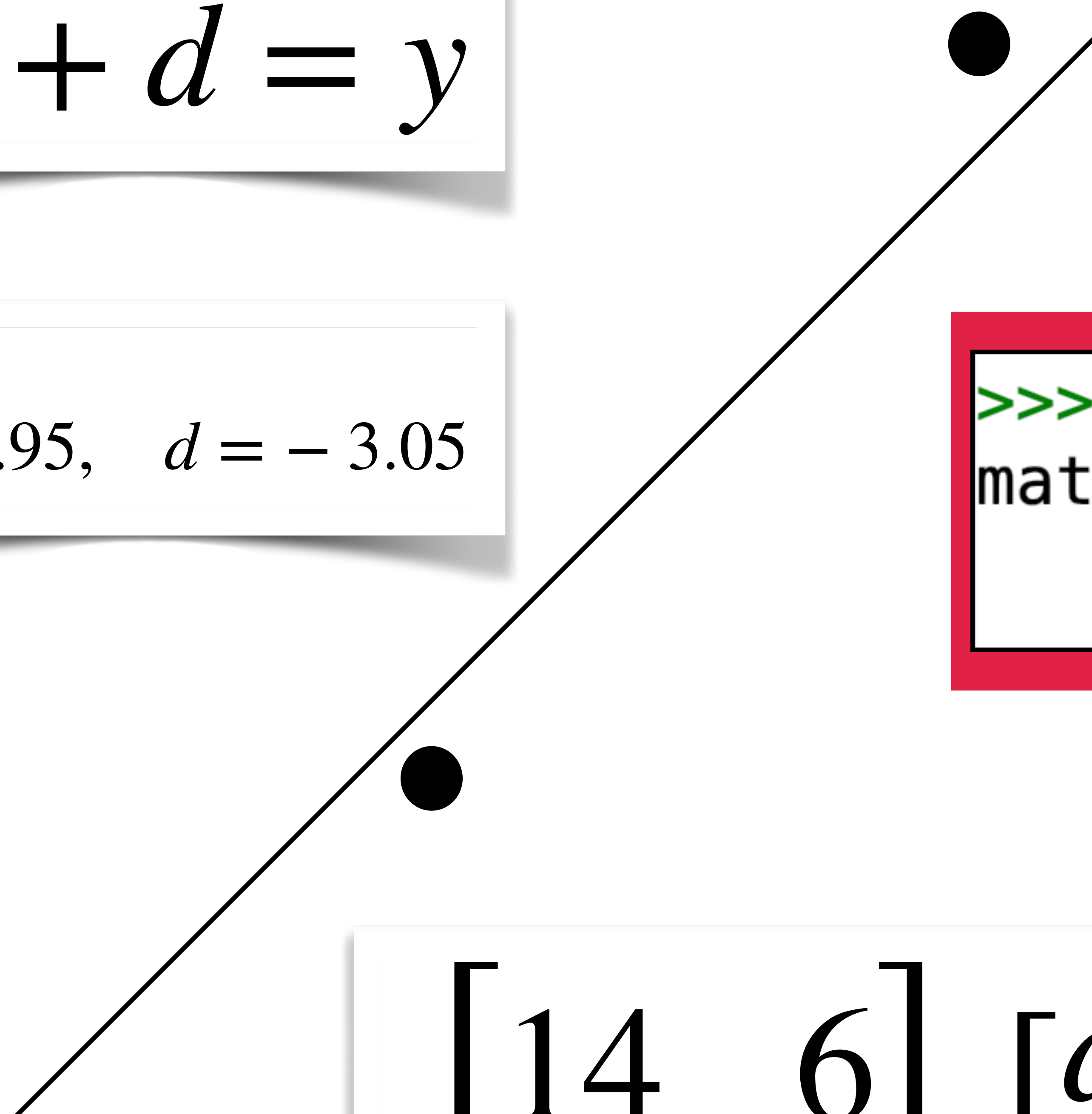
```
>>> invA
matrix([[ 0.2, -0.3],
        [-0.3,  0.7]])
```

$$inv(\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix})\begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$

```
>>> ans
matrix([[ 3.95],
        [-3.05]])
```

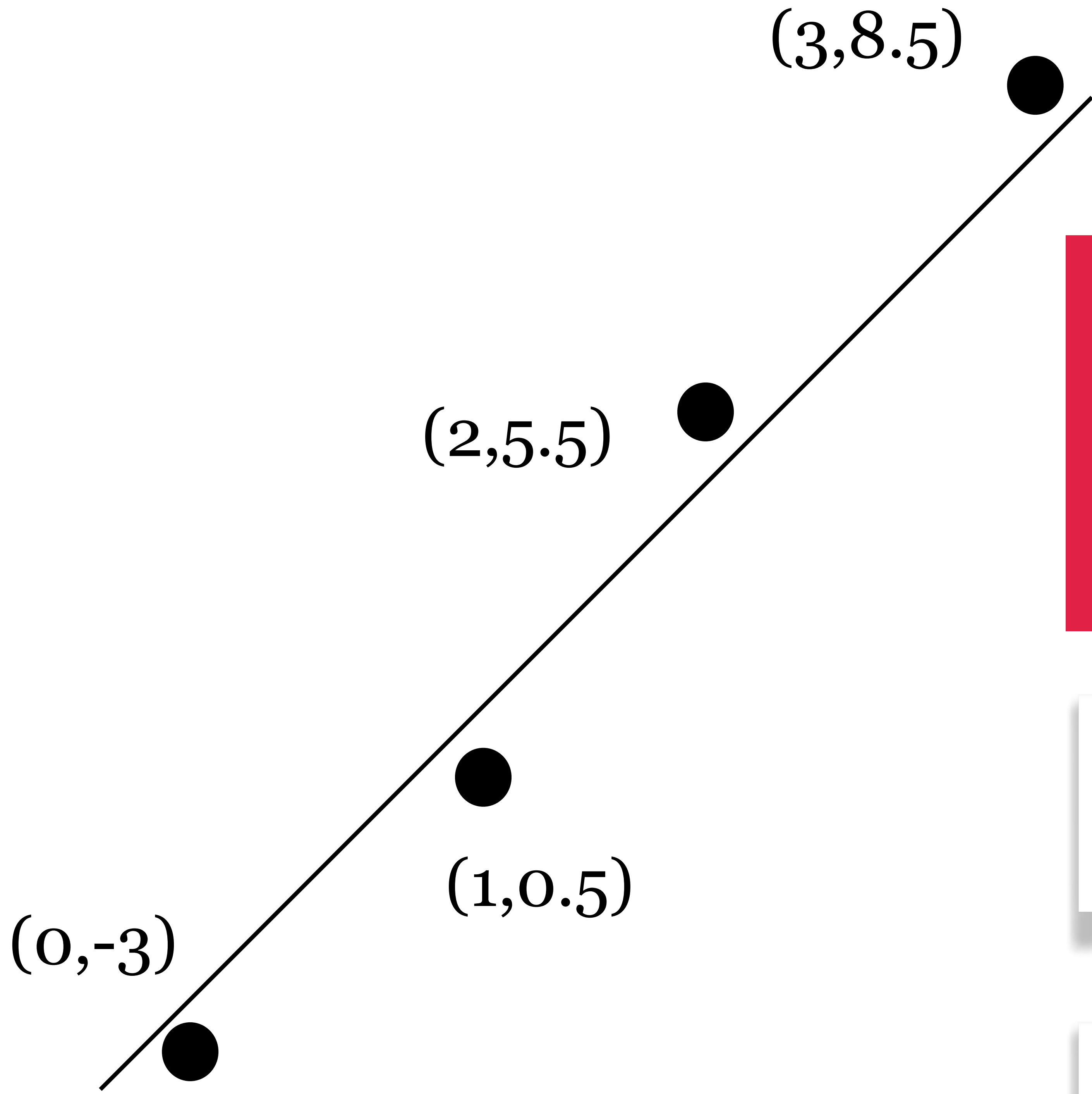
$$ax + d = y$$

$$a = 3.95, \quad d = -3.05$$



```
>>> ans
matrix([[ 3.95],
        [-3.05]])
```

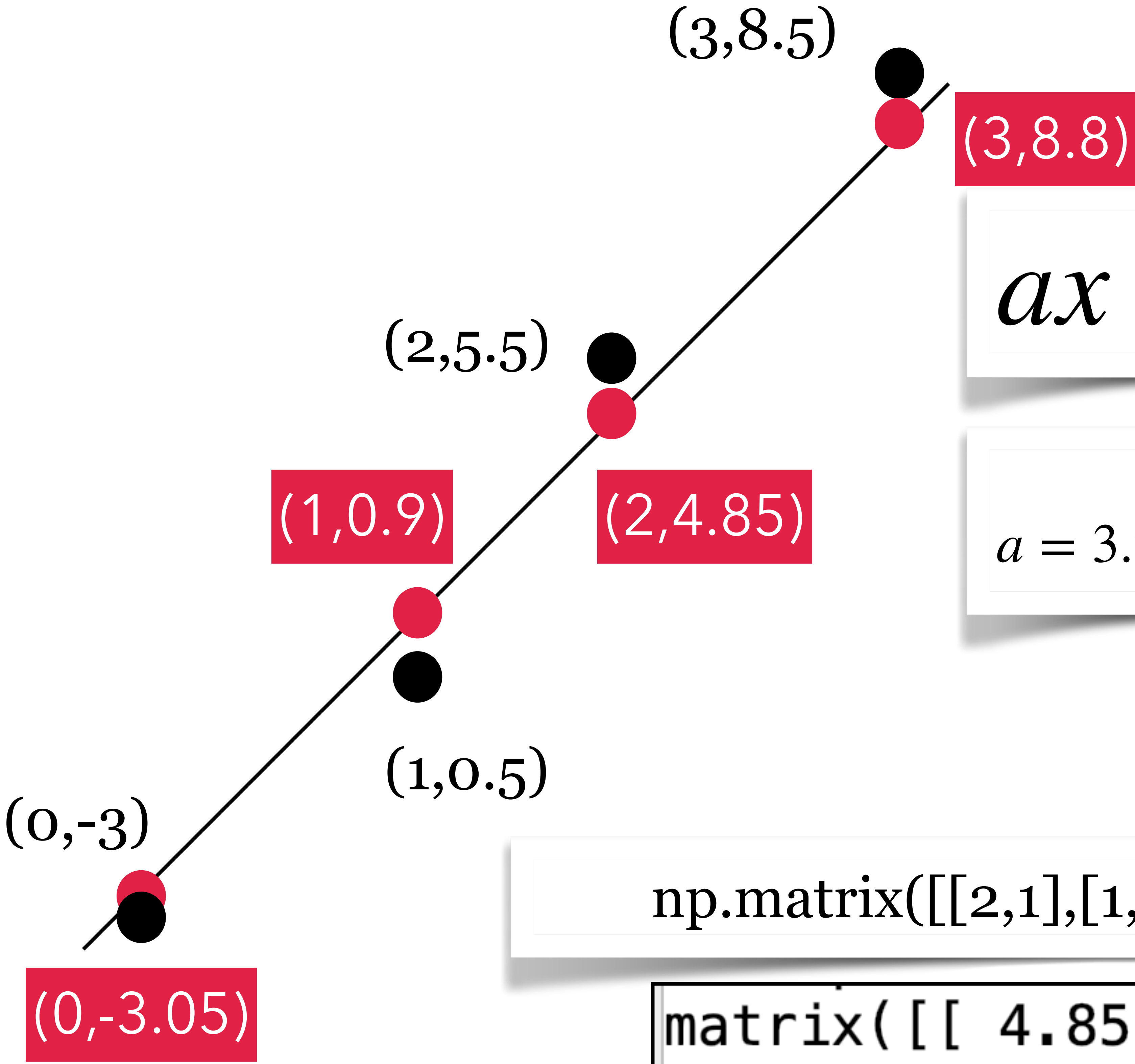
$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 37 \\ 11.5 \end{bmatrix}$$



四點不共線
如何求出最好的一條線，
描述這四點呢？

$$ax + d = y$$

$$a = 3.95, \quad d = -3.05$$

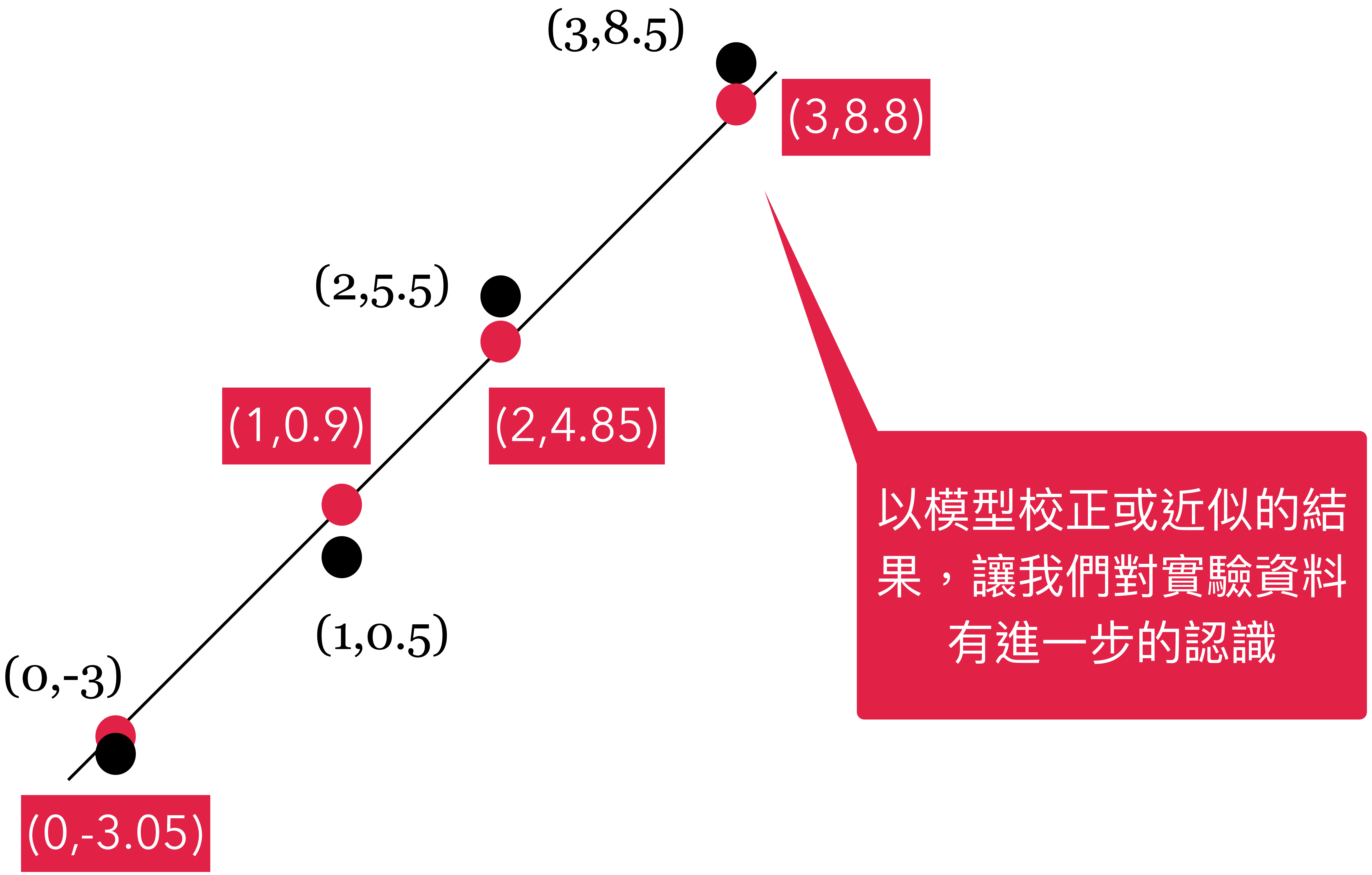


$$ax + d = y$$

$$a = 3.95, \quad d = -3.05$$

np.matrix([[2,1],[1,1],[3,1],[0,1]]) @ ans

```
matrix([[ 4.85],
        [ 0.9 ],
        [ 8.8 ],
        [-3.05]])
```


(3,8.5)
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(0,-3)
(0,-3.05)
以模型校正或近似的結果，讓我們對實驗資料有進一步的認識

方法

$$\underbrace{\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{A^t} \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 3 & 1 \end{bmatrix}}_{A} \begin{bmatrix} a \\ d \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}}_{A^t} \underbrace{\begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}}_{b}$$

$$B = A^t A \qquad A^t \qquad b$$

$$\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

B^{-1} A^t b

$$\begin{bmatrix} a \\ d \end{bmatrix} = inv(\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix}) \begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

$$B^{-1} \qquad A^t \qquad b$$

$$\begin{bmatrix} a \\ d \end{bmatrix} = inv\left(\begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 & 0 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 5.5 \\ 0.5 \\ -3 \\ 8.5 \end{bmatrix}$$

```
B = AT @ A
invB = inv(B)
ans = invB @ AT @ b
```

步驟一：匯入套裝

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步驟二：將資料以矩陣A向量b表示

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```

步驟三：求轉置矩陣

```
AT = np.matrix.transpose(A)
```

步驟四：求最佳參數

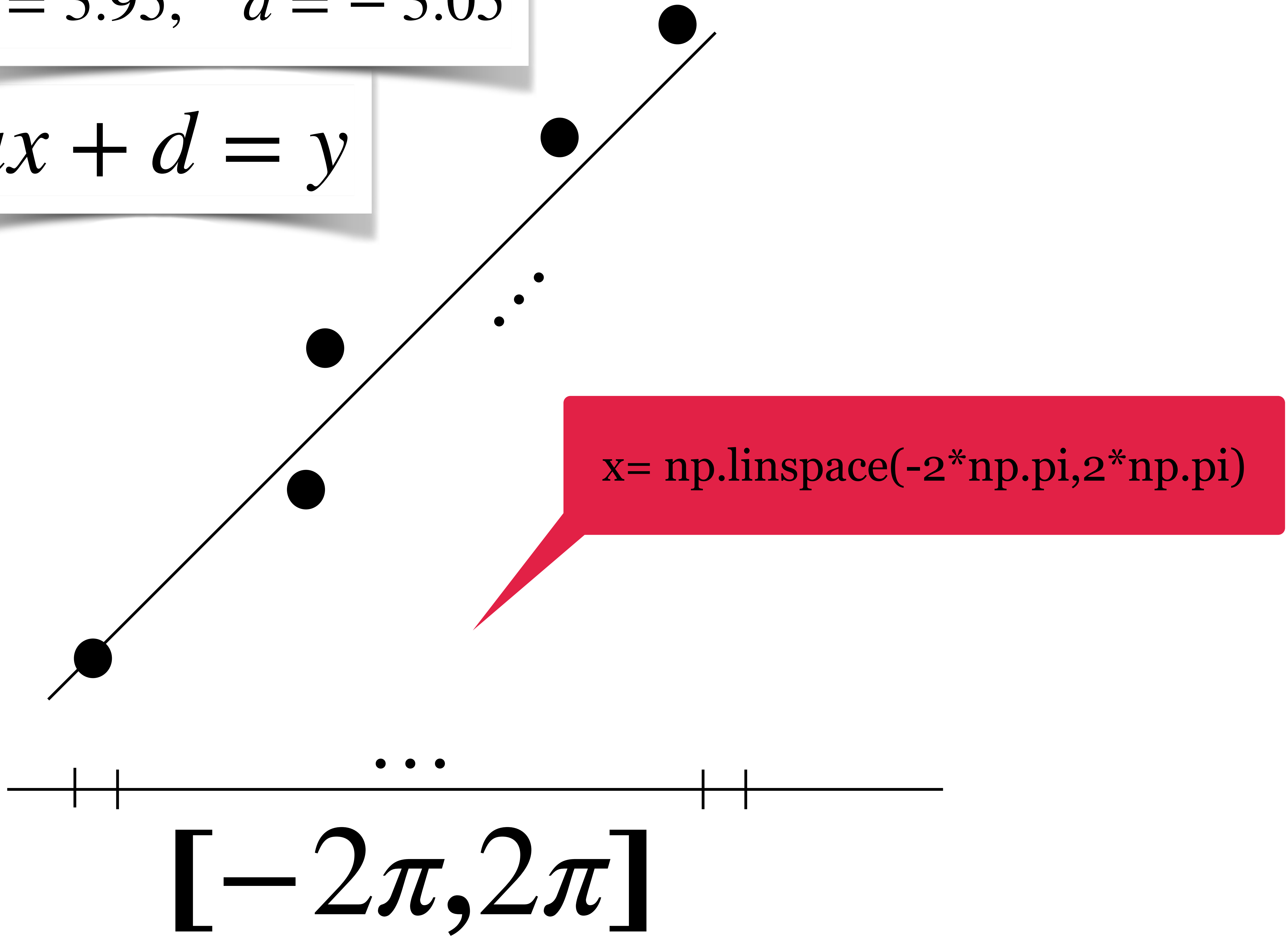
```
B = AT @ A
invB = inv(B)
ans = invB @ AT @ b
```

製作模擬的實驗資料

可以擴充到更多點？
在$[-2\pi, 2\pi]$的區間取50點
對應到直線上，取值加雜訊

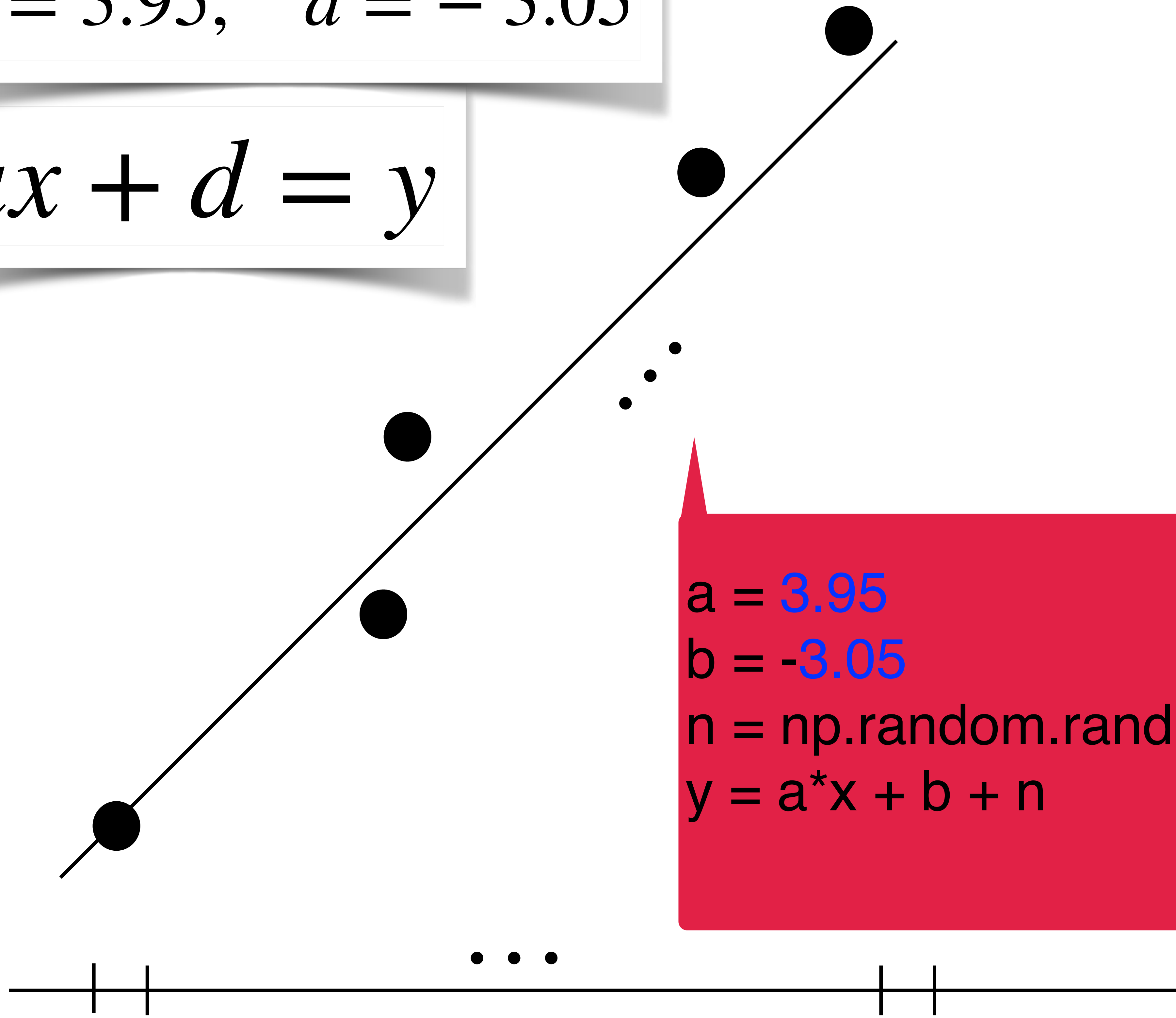
$$a = 3.95, \quad d = -3.05$$

$$ax + d = y$$



$$a = 3.95, \quad d = -3.05$$

$$ax + d = y$$



```
a = 3.95
b = -3.05
n = np.random.rand(1,50)
y = a*x + b + n
```

$$[-2\pi, 2\pi]$$

```
x= np.linspace(-2*np.pi,2*np.pi)
```

```
a = 3.95
b = -3.05
n = np.random.rand(1,50)-0.5
x = np.linspace(-2*np.pi,2*np.pi)
y = a*x + b + 0.1 * n
```

```
vx = np.matrix(x)
vy = np.matrix(y)
```

```
A = np.hstack((np.transpose(vx), np.ones((50,1))))
b = np.transpose(vy)
```

如何使用步驟三、步驟四求直線參數？